INTERMEDIATE ALGEBRA/MATH 64

SHANNON GRACEY

- π 100 POINTS POSSIBLE
- π YOUR WORK MUST SUPPORT YOUR ANSWER FOR FULL CREDIT TO BE AWARDED
- π YOU MAY USE A SCIENTIFIC CALCULATOR
- π PROVIDE EXACT ANSWERS UNLESS OTHERWISE INDICATED

ONCE YOU BEGIN THE EXAM, YOU MAY NOT LEAVE THE PROCTORING CENTER UNTIL YOU ARE FINISHED…THIS MEANS NO BATHROOM BREAKS!

NAME ____

MATH 64/GRACEY/INTERMEDIATE ALGEBRA NAME Key

EXAM 3/~~200~~ 102 POINTS POSSIBLE

This exam was curved – points possible changed to 95.

<u>CREDIT WILL BE AWARDED BASED ON WORK SHOWN. THERE WILL BE NO CREDIT FOR GUESSING. PLEASE PRESENT YOUR WORK IN AN ORGANISED, EASY TO READ FASHION.</u>

1. (8 POINTS) Find the distance between the pair of points $(-1,0)$ and $(-3,-8)$. If necessary, express the answer in **<u>simplified radical form</u>**.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - {}_1)^2}$$

$$d = \sqrt{(-3-(-1))^2 + (-8-0)^2}$$

$$d = \sqrt{(-2)^2 + (-8)^2}$$

$$d = \sqrt{4+64}$$

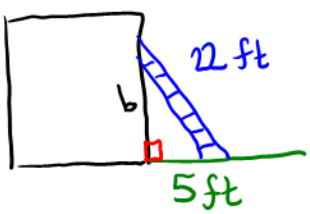
$$d = \sqrt{68}$$

$$d = \sqrt{(4)(17)}$$

$$d = 2\sqrt{17} \text{ units}$$

Distance: $\boxed{2\sqrt{17} \text{ units}}$

2. (12 POINTS) No credit will be awarded for guessing. A 22 foot ladder is leaning against a building, with the base of the ladder 5 feet from the building. How high up on the building will the top of the ladder reach? You may round to the nearest tenth of a foot, if necessary.



$$a^2 + b^2 = c^2$$

$$(5)^2 + (b)^2 = (22)^2$$

$$25 + b^2 = 484$$

$$b^2 = 459$$

$$b = \sqrt{(9)(51)}$$

$$b = 3\sqrt{51} \text{ ft}$$

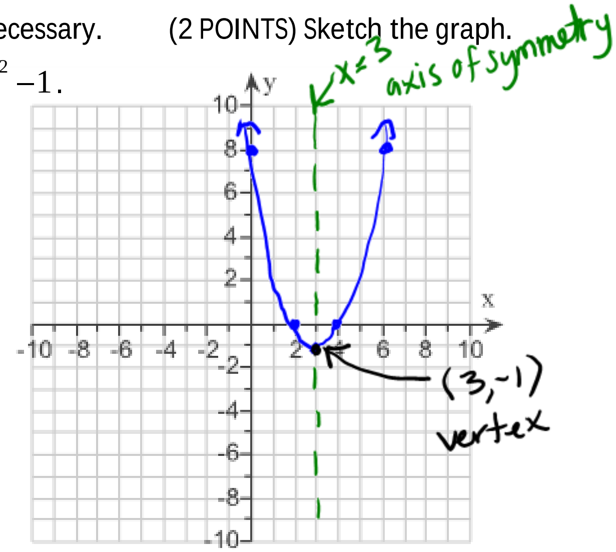
$$b \approx 21.4 \text{ ft}$$

The top of the ladder rests approximately at a height 21.4 ft.

3. (14 POINTS) You may round the intercepts to the nearest tenth, if necessary.

Use the vertex and intercepts to sketch the graph of $f(x) = (x-3)^2 - 1$.

(2 POINTS) Sketch the graph.



(4 POINTS) Vertex: $\boxed{(3,-1)}$

$f(x) = a(x-h)^2 + k \rightarrow a=1, h=3, k=-1$

$f(x) = 1(x-3)^2 + (-1)$

opens upward since $a>0$.

vertex: (h,k)

Vertex: $(3,-1)$

(4 POINTS) *x*-intercepts: $\boxed{(2,0) \text{ and } (4,0)}$

$f(x) = (x-3)^2 - 1$

$0 = (x-3)^2 - 1$

$+1 \qquad +1$

$\sqrt{1} = \sqrt{(x-3)^2}$

$\pm 1 = x-3$

$x-3=-1$ or $x-3=1$

$x=2 \qquad x=4$

(1 POINT) Axis of symmetry: $\boxed{x=3}$

$x=h$

(1 POINT) *y*-intercept: $\boxed{(0,8)}$

$f(x) = (x-3)^2 - 1$

$f(0) = ((0)-3)^2 - 1 \rightarrow f(0) = 9-1 \rightarrow f(0) = 8$

(2 POINTS) Domain in interval notation: $\boxed{(-\infty, \infty)}$

(2 POINTS) Range in interval notation: $\boxed{[-1, \infty)}$

(x_1, y_1) (x_2, y_2)

4. (4 POINTS) Find the midpoint of the line segment with endpoints $(-2,4)$ and $(9,1)$.

midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-2+9}{2}, \frac{4+1}{2}\right)$

$= \left(\frac{7}{2}, \frac{5}{2}\right)$

Midpoint: $\boxed{(7/2, 5/2)}$

5. (10 POINTS) Solve the following equation using the method of your choice. Give exact answers, using radicals and i as needed.

$7x^2 = -4x - 2$

$+4x+2 \quad +4x+2$

$7x^2 + 4x + 2 = 0$

$ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 7, b = 4, c = 2$

$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(7)(2)}}{2(7)}$$

$$x = \frac{-4 \pm \sqrt{16 - 56}}{14}$$

$$x = \frac{-4 \pm \sqrt{-40}}{14}$$

$$x = \frac{-4 \pm \sqrt{(4)(-1)(10)}}{14}$$

$$x = \frac{-4 \pm 2i\sqrt{10}}{14}$$

$$x = \frac{\cancel{2}^{1}(-2 \pm i\sqrt{10})}{\cancel{14}\,7}$$

$$x = \frac{-2 \pm \sqrt{10}\, i}{7}$$

$$\boxed{\left\{ \frac{-2 - \sqrt{10}\, i}{7}, \frac{-2 + \sqrt{10}\, i}{7} \right\}}$$

6. (10 POINTS) Solve. Give exact answers, using radicals and i as needed.

$x^4 - x^2 - 6 = 0$

$(x^2)^2 - (x^2) - 6 = 0$

$u^2 - u - 6 = 0$

$(u-3)(u+2) = 0$

$u-3=0$ or $u+2=0$

$u=3$ $\quad$ $u=-2$

Let $u = x^2$, so $x = \pm\sqrt{u}$

$u=-2$

$x = \pm\sqrt{-2}$

$x = \pm\sqrt{2}\,i$

$u=3$

$x = \pm\sqrt{3}$

$\boxed{\{-\sqrt{2}\,i, -\sqrt{3}, \sqrt{2}\,i, \sqrt{3}\}}$

7. (8 POINTS) Evaluate the following expressions without using a calculator. Each problem is worth 3 points.

a. $\log_2 16 =$ $\boxed{4}$

b. $\log_{49} 7 =$ $\boxed{\frac{1}{2}}$

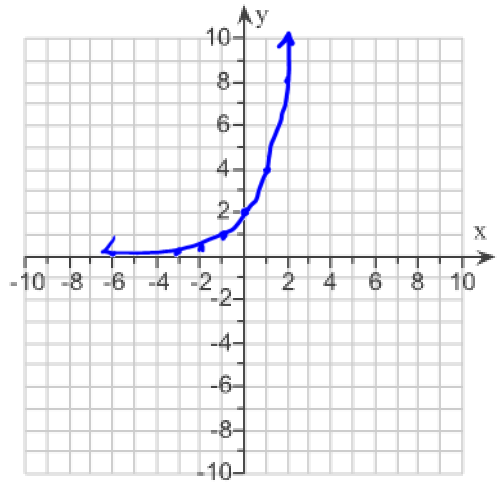
c. $\log 10^{20} =$ $\boxed{20}$

d. $\ln e =$ $\boxed{1}$

8. (8 POINTS) Consider the exponential function $y = 2^{x+1}$.

a. (6 POINTS) Sketch the graph by hand.

x	$y=2^{x+1}$	(x,y)
-3	$y=2^{-3+1}=\frac{1}{4}$	$(-3,\frac{1}{4})$
-2	$y=2^{-2+1}=\frac{1}{2}$	$(-2,\frac{1}{2})$
-1	$y=2^{-1+1}=1$	$(-1,1)$
0	$y=2^{0+1}=2$	$(0,2)$
1	$y=2^{1+1}=4$	$(1,4)$



b. (1 POINT) The domain in interval notation is: $\boxed{(-\infty,\infty)}$

c. (1 POINT) The range in interval notation is: $\boxed{(0,\infty)}$

9. (4 POINTS) Evaluate the expression without using a calculator. Show all work!

$\log_2(\log_3 81) = \log_2(4)$

$= \boxed{2}$

10. (10 POINTS) **<u>Solve by completing the square</u>**. Give exact answers, using radicals and i as needed.

$x^2 - 8x - 4 = 0$

$\quad +4 \quad +4$

$x^2 - 8x + (-4)^2 = 4 + (-4)^2$

$(x-4)^2 = 4 + 16$

$\sqrt{(x-4)^2} \quad \sqrt{20}$

$x-4 = \pm\sqrt{20}$

$x-4 = \pm\sqrt{(4)(5)}$

$x-4 = \pm 2\sqrt{5}$

$x = 4 \pm 2\sqrt{5}$

$\boxed{\{4-2\sqrt{5},\ 4+2\sqrt{5}\}}$

11. (12 POINTS) The profit, $P(x)$, generated after producing and selling x units of a product is given by the function $P(x)=R(x)-C(x)$, where R and C are the revenue and cost functions, respectively. A local sandwich store has a fixed weekly cost of \$545.00, and variable costs for making a roast beef sandwich are \$0.60.

a. (2 POINTS) Let x represent the number of roast beef sandwiches made and sold each week. Write the weekly cost function, C, for the local sandwich store.

$$\boxed{C(x) = 0.60x + 545.00}$$

b. (4 POINTS) The function $R(x)=-0.001x^2+3x$ describes the money that the local sandwich store takes in each week from the sale of x roast beef sandwiches. What is the weekly profit function?

$P(x) = R(x) - C(x)$

$P(x) = (-0.001x^2 + 3x) - (0.60x + 545.00)$

$P(x) = -0.001x^2 + 2.40x - 545.00$

$$\boxed{P(x) = -0.001x^2 + 2.40x - 545.00}$$

c. (6 POINTS) Use the store's profit function to determine the number of roast beef sandwiches it should make and sell each week to maximize profit, and find the maximum weekly profit.

We need to find the vertex (h,k)

$a=-0.001$, $b=2.40$

$h = -\frac{b}{2a}$

$h = -\frac{2.40}{2(-0.001)}$

$h = \frac{2.40}{0.002}$

$h = 1200$

The store needs to make and sell 1200 roast beef sandwiches to maximize profit.

$k = P(1200) = -0.001(1200)^2 + 2.40(1200) - 545.00$

$P(1200) = 895$. The maximum profit is \$895.00

The maximum weekly profit is $\boxed{\$895}$ when $\boxed{1200}$ sandwiches are sold.